

# Look at the holographic universe from a moving frame and Casimir energy

V I Dokuchaev and Yu N Eroshenko

Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia

E-mail: dokuchaev@ms2.inr.ac.ru, eroshenko@ms2.inr.ac.ru

**Abstract.** We consider the holographic universe from the point of view of the observer moving with respect to CMB frame. The Lorentz noninvariance of holographic approach is demonstrated. This is because the UV cutoff obeys the local Lorentz invariance while the IR cutoff does not. The later cutoff is settled by the cosmological horizon which deforms for the moving observer. The analogy in the cosmological Casimir energy is discussed by taking in mind that the virtual particle Compton wavelengths cannot exceed the horizon size, as causality dictates.

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## 1. Introduction

The concept of holographic dark energy has the beginning in the holographic principle [1], [2]. In the work [3], inspired by the Bekenstein limit for the maximum entropy  $S/E < 2\pi L$  [4], the connection between UV and IR cutoffs was settled. Then [5] proposed the concrete realization of a holographic principle in cosmology (FS - conjecture), which was modified in many subsequent works. It was realized in [6], [7] that the holographic principle requires more exact definition of concept of degree of freedom. By means of the formalism of [3] and guess of the use of the future event horizon as IR cutoff, the [8] introduced the rather reliable model of holographic Dark Energy (DE) with correct equation of state. And the whole holographic approach was advanced in plenty of works until the recent explanation of gravity in entropic terms [9].

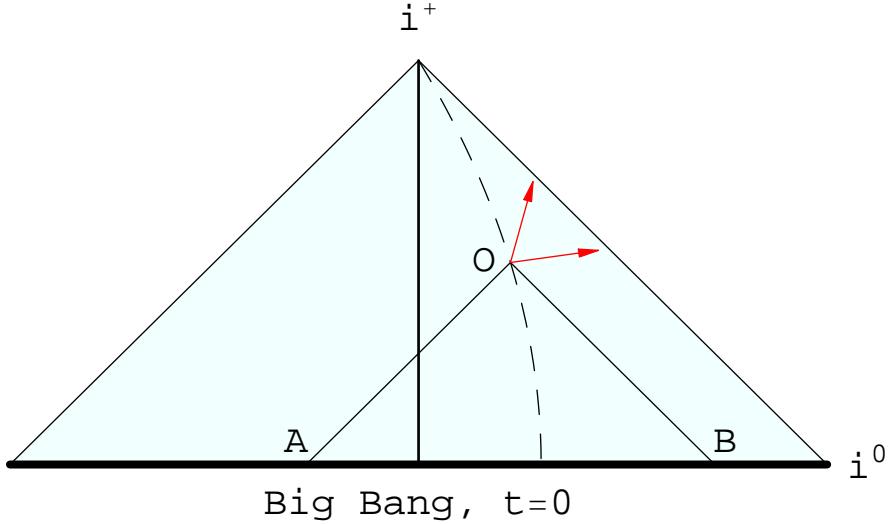
The Casimir effect [10] in the standard statement of the problem consists in an attraction of two metal plates separated by the small distance  $l$ . This phenomenon arises because the zero vacuum fluctuations modes in space between the plates can have only wavelengths  $\lambda \leq 2l$ . Therefore the density of the zero energy fluctuations (Casimir energy) between the plates is less, than outside, and it causes their attraction. In a number of works the Casimir energy was considered in the cosmological context starting from Ya.B. Zeldovich proposal [11]. In particular, there were attempts to explain the cosmological constant and DE by the zero fluctuations of vacuum fields. The resulting energy density exceeds the observed value by many orders of magnitude, and some procedures of regularization are required.

In these notes we consider both the holographic DE and Casimir energy from the point of view of a moving observer. We show that the cosmological horizon deforms (becomes elongated) for this observer. This leads to some consequences as for holographic DE (because it includes the concept of boundary surface in the entropy bound) and for Casimir energy due to softer cutoff of zero fluctuations in the direction of motion in comparison with transverse directions.

We refer as the “system at rest” the system in which the CMB has no dipole anisotropy. In relation to such system the Sun moves with velocity  $370 \text{ km s}^{-1}$  towards constellation of the Lion. This preferential frame  $K$  was called “C-frame” in the works [12]. Another definition of this frame based on the isotropy of Hubble flow, this is also the frame in which the baryon matter is locally at rest. In differential geometry C-frame lies at the hypersurface, perpendicular to the lines of time. Probably, this frame was selected at the epoch of reheating after inflation due to spontaneous symmetry breaking onto the particular quantum vacuum state [12].

## 2. Cosmological horizon for moving observer

Let us consider the two systems of coordinates: system  $K$  which is at rest with respect to CMB and system  $K'$ , moving freely with respect to  $K$  in the positive direction of axes X, and we suppose that at the moment  $t = t_0$  the origins of the systems  $K$  and  $K'$



**Figure 1.** Schematic Carter-Penrose diagram of Freedman-Robertson-Walker universe in the  $t - x$  plane. Segment AB is the particle horizon of the co-moving observer  $O$  at  $t = t_0$ . The arrows show the tetrad of the observer  $O'$ , having Lorentz boost at the moment  $t_0$ . For the boosted observer  $O'$  the physical size of AB is larger in comparison with the measurements of the co-moving observer  $O$ .

coincide. We use the operational definition of cosmological horizon (particle horizon) proposed by Rindler as “a frontier between things observable and things unobservable” [13]. The particle horizon, i.e. the maximum causally connected region, in  $K$  is a sphere of the radius  $R_H = a(t_0) \int_0^{t_0} dt/a(t)$ , where  $a(t)$  is the scale factor of the universe with metrics

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \quad (1)$$

and we restrict ourself only by the flat space. The function  $a(t)$  depends on the details of cosmology. Schematic Carter-Penrose diagram of Freedman-Robertson-Walker universe in the  $t - x$  plane is shown at Fig. 1.

Imagine for a moment that we are in Minkovski space which appeared at  $t = 0$  by the clock of the system  $K$ . In this toy model the observer in  $K$  at  $t = t_0$  has the spherical horizon with radius  $R_H = ct_0$ . The points of the hypersurface  $t = 0$  is not simultaneous in the moving system  $K'$ , they have different  $t'$  at different points in  $X'$  direction, as it is obvious from the Lorentz transformations. Let us consider the converging spherical light wave that focused at the common beginnings of  $K$  and  $K'$  at the moment  $t = t' = t_0$  and put that this wave was emitted near the moment  $t = 0$ . For the observer in the system  $K$  the emitters are on the spherical surface  $S$  with radius  $R_H$ . For the observer in  $K'$  the corresponding surface  $S'$  represents a surface of ellipsoid with semiaxes  $\gamma R_H$ ,  $R_H$  and  $R_H$ , where  $\gamma$  - is the Lorentz-factor. Note that the surface  $S'$  is elongated (not shortened for a moving observer!) because its boundary is the surface of simultaneity in

the system  $K$  (see also [14]). As the converging light front in the both systems remains spherical, it is clear that in the system  $K'$  atoms of the surface  $S'$  have radiated light not simultaneously, and atoms in  $+X'$  direction radiated first. Here the preference of the system  $K$  among all over inertial systems arises due to our choice of  $t = 0$  as the time birth by the clocks of  $K$ . The universe's expansion somehow complicates these arguments, but the effect of horizon deformation is roughly the same.

Now we return to the real universe. It's difficult to construct the rigid system  $K'$  in the whole expanding universe. Instead, we define the flexible moving system by the following way. Let us consider the set of test (dust) particles all moving freely in the positive direction of axes  $X$  with equal velocities  $u^\mu = dx^\mu/ds$ . Their physical components  $U^\mu = a(t)u^\mu \propto 1/a(t)$ ,  $U^1 = U$ ,  $U^2 = U^3 = 0$  and the ordinary velocity is expressed as  $v^2 = U^2/(1 + U^2)$ . Let us consider some event in  $K'$  and the test particle, which coincides with this event. We put the time of the event equals to the proper time of the particles passed from the  $K$  and  $K'$  coincidences at  $t = t_0$ . So we calculate the interval along the world line of the test particle:

$$d\tau^2 = ds^2 \Big| \text{ along the w.l.} = \frac{dt^2}{1 + U^2}. \quad (2)$$

As the space coordinate we choose the coordinate of the test particle under consideration in  $K$ -system at  $t = t_0$ :

$$x' = x - \int_{t_0}^t \frac{U dt}{\sqrt{1 + U^2}}, \quad (3)$$

and the transverse coordinates are  $y' = y$  and  $z' = z$ . These space coordinates are therefore the Lagrange coordinates of the test particles in  $K'$ , but numbered by the coordinates of  $K$  at the moment  $t = t_0$ . In the system  $K'$  with the above coordinates transformations the interval takes the form

$$ds^2 = d\tau^2 - 2aUd\tau dx' - a^2(dx'^2 + dy'^2 + dz'^2). \quad (4)$$

In the metrics (4) the ideal fluid energy density is transformed as

$$\varepsilon' = T_{0'0'} = \gamma^2(\varepsilon + pv^2), \quad (5)$$

this coincides with the local Lorentz transformation law [15] (§ 35), and it's interesting that the phantom matter gets the negative energy density.

The light cone obeys the differential equation  $ds^2 = 0$ . In the  $Y'$  and  $Z'$  directions the light cone and the horizon size are the same as in the original  $K$ -system. In the  $X'$  direction the light front moves according to the expressions

$$x'_\pm = \int_0^\tau d\tau (-U \pm \gamma)/a(t) + C_\pm = \int_{t_0}^t dt (-v \pm 1)/a(t), \quad (6)$$

where the signs  $\pm$  mark the two direction of the light propagation, and the constants of integration  $C_\pm$  in the last expression were fixed by the conditions  $x'_\pm(t = t_0) = 0$ . The

full coordinate size of the particle horizon in  $X'$  direction (distance between the events A and B at Fig. 1) is

$$\Delta x' = x'_-(0) - x'_+(0) = 2 \int_0^{t_0} dt/a(t) = 2R_H(t_0)/a(t_0), \quad (7)$$

For the future event horizon the analogous expression reads as

$$\Delta x'_F = x'_+(t = \infty) - x'_-(t = \infty) = 2 \int_{t_0}^{\infty} dt/a(t) = 2R_F(t_0)/a(t_0), \quad (8)$$

Therefore the coordinate sizes of the horizons are the same in  $K$  and  $K'$ :  $\Delta x' = \Delta x$ ,  $\Delta y' = \Delta y$  and  $\Delta z' = \Delta z$ . But the physical size in  $K'$  is equal to [15] (Chapter X, §84)

$$dl'^2 = \left( -g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}} \right) dx^\alpha dx^\beta = a^2(\gamma^2 dx'^2 + dy'^2 + dz'^2), \quad (9)$$

where  $\gamma(t) = (1 - v^2(t)/c^2)^{-1/2}$ . The expression (9) taken at  $t = \text{const}$  means that the horizons sizes (both particle and future event horizons) are larger in  $K'$ -system in  $X'$  direction in  $\gamma(t)$  times in comparison with  $K$ -system.

An observer in  $K'$  sees the bulk motion of all the matter (except for the test particles), which is at rest in  $K$ . This situation is similar to the anisotropic cosmological models with bulk motion instability [16] (Chapter 19, §5), but the systems in both cases are differ in construction. We really obtain from (4) the anisotropic cosmology with diagonal metrics if we introduce the new time variable  $dt'' = d\tau - aUdx'$ , which corresponds to the clock synchronization procedure [15]. Indeed, in the system (4) the simultaneity of any two clocks corresponds to the the simultaneity of the clocks in the original  $K$ -system, because the  $\tau$  is the function of  $t$  only. But the clocks in the moving system  $K'$  are not physically synchronized between themselves.

One may consider an observer that originally at rest with respect to CMB frame  $K$ , accelerated for some time interval  $\delta t$ , and then moves freely with  $K'$ . From the point of view of this observer the causal region elongated from sphere to ellipsoid. In the next sections we consider the consequences of the horizon deformation. Note again that the nonsphericity of the causally-connected area in  $K'$  is the result of the existence of preferred system  $K$  (C-frame) connected with CMB in the expanding universe.

To avoid confusion with the problem of the apparent shape of a relativistically moving sphere, we must note that the horizon deformation under consideration is defined by causality reasons, in contrast with aberration effect [17], which is purely local phenomenon. Therefore the discussion between [14] and [18] is not applicable to our case, in particularly because we consider an observer *inside* the surface of a horizon.

### 3. Holographic DE in a moving frame

With the saturated bound from [3] (Eq. (2)) the holographic DE density was presented in [8] as

$$\rho_{\text{DE}} \simeq \frac{3}{8\pi GL^2}, \quad (10)$$

where the choice of IR cutoff at the particle horizon  $L \simeq R_H$  gives correct energy density, and the equality of  $L$  with the future event horizon radius  $R_F$  reproduces also the correct equation of state of DE.

The surface area of the deformed horizon in the limit  $\gamma \gg 1$  can be expressed as

$$A \simeq \pi^2 R_H^2 \gamma. \quad (11)$$

(or one can use here and later  $R_F$  instead of  $R_H$ ). The effective scale  $L$  in the universal entropy bound  $S/E < 2\pi L$  is defined as [4]

$$L = (A/4\pi)^{1/2}, \quad (12)$$

where  $A$  is the area of surface which circumscribes the system. It was proposed originally [4] that the surface must be (quasi) spherical, and only largest scale of a system plays the role in the universal entropy bound. The largest scale defines only the minimum frequency  $\omega_1 > \pi/R$  of the system. But the calculation of entropy and energy out of equilibrium requires the knowledge of the full geometry and the mode composition. Really, largest modes just go into horizon and their thermalization at the cosmological scale during the Hubble time is doubtful. Let us use (12) as the effective scale even for highly nonspherical cosmological horizon viewed from the moving frame. Also we suppose that the holographic DE saturates the bound of [3] in the system  $K'$  just like in the  $K$ . From (11) and (12) we obtain for the effective scale  $L' \sim R_H \gamma^{1/2}$  and from (10) we have finally

$$\rho'_{\text{DE}} \simeq \rho_{\text{DE}} \gamma^{-1}, \quad (13)$$

i.e. the relativistically moving observer sees the very diminished holographic DE density. With the [4] choice of largest size  $L = R_H \gamma$  the effect would be even stronger  $\rho'_{\text{DE}} \simeq \rho_{\text{DE}} \gamma^{-2}$ .

Usual dependence on the UV cutoff energy scale  $\Lambda$  is  $\rho_{\text{DE}} \propto \Lambda^4$ . We may keep the statement that the microscopic physics relevant for the UV cutoff is Lorentz invariant. I.e.  $\Lambda$  and over microscopic quantities are the same in  $K$  and  $K'$ . This is also supported by the operational point of view [2]: the observer which accelerated from  $K$  to  $K'$  may carry the same apparatus. In both system  $K$  and  $K'$  the apparatus has the same high energy (high frequency) cutoff of its observational capability. This is in conflict with the above diminishing of the DE density. Therefore, we must conclude that the holography DE models are not Lorentz invariant as a consequence of the horizon deformation.

For the usual ideal fluid the transformation law is given by (5). The feature of the holographic DE model is the noncovariant change (13). Another problem is the rise of information due to surface area growth. Really, the number of bits at the holographic

screen S' is  $\sim A'/l_P^2$  increases during the acceleration of observer because the Plank scale  $l_P$  is invariant. We hypothesize that the above energy density transformation somehow related to the vacuum state change during the time interval  $\delta t$  of acceleration. Such a change is not surprising in view of Unruh effect, but the particular mechanism is desirable.

#### 4. Casimir energy example

Now we consider the influence of the horizon deformation on the Casimir energy. The density of the scalar field zero energy is

$$\varepsilon = \frac{1}{8\pi^3} \int_{k_{\min}}^{k_{\max}} d^3 k \sqrt{k^2 + m^2}. \quad (14)$$

First we consider the Casimir energy in the rest frame  $K$ . For the massless field  $\varepsilon = (k_{\max}^4 - k_{\min}^4)/8\pi^2$ , where  $k_{\min}$  and  $k_{\max}$  correspond to the IR and UV cutoffs, respectively. In the models of the closed or torus-topology universe the IR cutoff of Casimir energy is usually putted to the universe radius  $k_{\min} = 1/a$  (see [19] for example). We state that in the Fridman universe the  $1/k_{\min}$  must be equated to the horizon size  $R_H$  even if no boundary conditions are explicitly imposed. This is because the Compton length of the virtual particle can not exceed the maximum causal scale, i.e. the horizon scale. In the opposite case the particle is disintegrated and does not exist as a particle supplying the Casimir energy. Note that this is different to the case of real classical fields which may have Fourier modes larger then the horizon scale modes (stretched out by inflation).

The UV divergence of the Casimir energy (at  $1/k_{\max}$  scale) is regularized by some method and we suppose that this procedure is locally Lorenz invariant and does not influence the IR cutoff. Let us suppose, that in the (14) the regularized term with  $k_{\max}$  gives the universe's DE  $\varepsilon_v = \rho_{\text{DE}}$  at  $k_{\min} \rightarrow \infty$ . The correction due to horizon cutoff  $k_{\min} \sim R_H \sim t$  has the following form

$$\varepsilon = \varepsilon_v - \frac{1}{8\pi^2 t^4}. \quad (15)$$

The correction is larger in the early cosmological epochs and it is close to  $\varepsilon_v$  at  $t_v \sim (8\pi^2 \varepsilon_v)^{-1/4} \simeq 10^{-13}$  s, and the Hubble radius at the time is  $ct_v \simeq 3 \times 10^{-3}$  cm. But the radiation has the energy density greater then  $\varepsilon_v$  by many orders at that time. So, the effect seems unobservable now and has no practical influences except for the fine tuning cases then  $\varepsilon_v$  changes at early phase transitions. The finiteness of the horizon is important at the inflation stage, where the quantum fluctuations of fields are limited by  $H^{-1}$  scale that was taken into account in many calculations.

Returning to moving observer we see that in the nonspherical geometry  $\gamma > 1$  one must integrate in (14) separately at  $X'$  and transverse directions. This leads to the more complicate correction in (15), and the velocity-dependent part of this correction

in the limit  $\gamma \rightarrow \infty$  takes the form  $\delta\varepsilon \simeq k_{\max}^3/(12\pi^2\gamma R_H)$ , that is, the Casimir energy becomes velocity dependent.

One may expect some consequences in the case of Higgs field's  $\phi$  zero fluctuation. This field is “un-removable” from space, it gives mass to the particles like  $m = \alpha\langle\phi\rangle$ . Even in the rest frame  $K$  the horizon expansion gives tiny correction to the Higgs field and to the particles masses. The masses of particles change during their acceleration due to the change of the Higgs' field expectation value  $\langle\phi\rangle$ , because the mode composition of  $\phi$  inside the horizon vary in analogy with (14). The corresponding corrections to the energy-momentum dispersion relations for particles are similar to the corrections in the models with local Lorentz invariance violation.

## 5. Conclusion

In this notes we made assumptions in the spirit of holographic DE models and came to the Lorentz noninvariance of holographic cosmology. Most striking conclusion is the diminishing of DE density (13) in the relativistic frame. On the over side, if the DE is nevertheless invariant it would be interesting to find the lame arguments in the above reasoning and to formulate the more correct method of considering the holographic universe from a moving frame.

As for Casimir effect, the phenomena in  $K$  and  $K'$  proceed equally only at the identical initial and boundary conditions in these systems. That is, the only local Lorentz-invariance does not guarantee equivalence of the phenomena. In our case the local Lorentz-invariance is in place, but the Casimir effect brings different boundary conditions for  $K$  and for moving  $K'$  observers. Namely, in moving system  $K'$  the size of horizon is larger in a proportion to the Lorentz-factor  $\gamma$ , and therefore the quantum fields are subjected to softer IR cutoff.

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